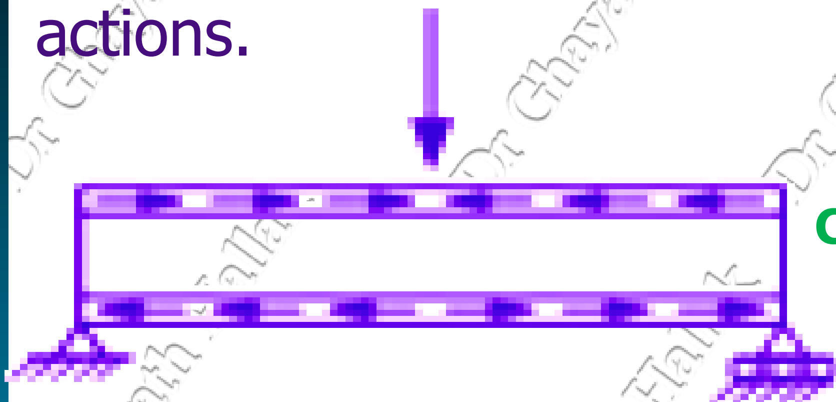
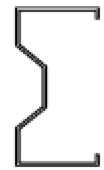
Design of
Restrained Beams

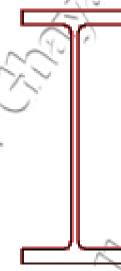
Beams

Beams are structural members which transfer the transverse loads they carry to the supports by bending and shear actions.

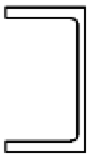




Cold-formed section

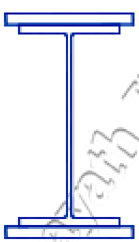


Universal beam

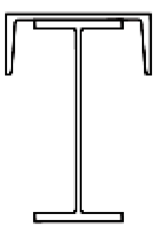


Rolled steel channel

A variety of section shapes and beams types may be used depending on the magnitude of loading and the span.



Compound beam



Crane beam

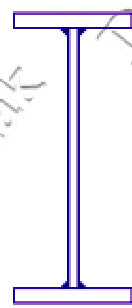
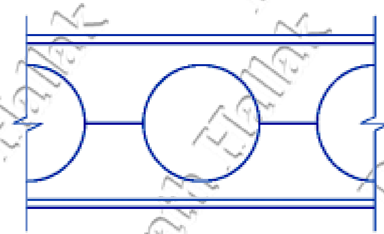
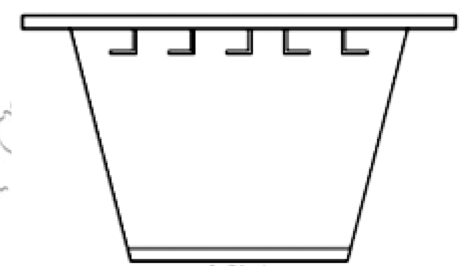


Plate girder



Cellular beam



Box girde

Typical usages of different forms of beam

Beam type	Span range (m)	Notes
Angles	1-6	Used for roof purlins, sheeting rails, etc. where only light loads have to be carried
Cold-formed	2-8	Used for roof purlins, sheeting rails, etc. where only light loads have to be carried
Sections Rolled sections: UBs, IPEs, UCs, HEs	1-30	Most frequently used type of section; proportioned to eliminate several possible modes of failure
Open web joists	4-40	Prefabricated using angles or tubes as chords and round bar for the web diagonals, used in place of rolled sections
Cellular beams	6-60	Used for long spans and/or light loads; depth of rolled section increased by 50%; web openings may be used for services etc.

Beams

Typical usages of different forms of beam

Beam type	Span range (m)	Notes
Compound sections	5-30	Used when a single rolled section would not provide sufficient resistance
Plate girders	10-100	Made by welding 3 plates, often automatically, with web depths up to 3-4 m; may need stiffening
Trusses	10-100	Fabricated from angles, tubes or, if spanning large distances, rolled sections
Box girders	15-200	Fabricated from plate, usually stiffened; used for overhead travelling cranes and bridges due to good torsional and transverse stiffness properties

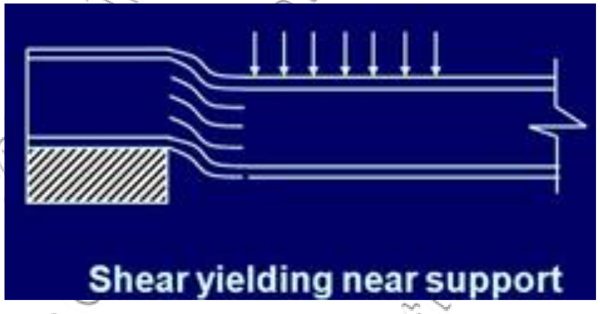
A beam is considered **restrained** (lateral-torsional buckling cannot develop) if:

- The cross-section of the beam is bent about its minor z axis
- The beam is laterally restrained by means of secondary steel members, by a concrete slab or any other method that prevents lateral displacement of the compressed parts of the cross section
- Closely spaced bracing is provided making the slenderness of the weak axis low, $\lambda_{LT} < 0.2$ (or 0.4 in some cases)
- The cross section of the beam has high torsional stiffness and similar flexural stiffness about both principal axes of bending as, for example, CHS, SHS, circular or square bar

Assessment of the restrained beam can be based just on the cross-section resistance $M_{c,Rd}$

Shear resistance $V_{c,Rd}$

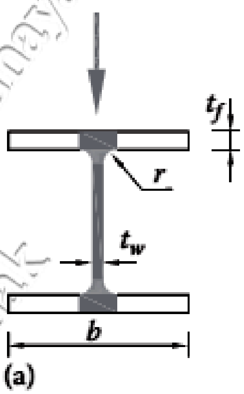
EN 1993-1-1: 2005- 6.2.6



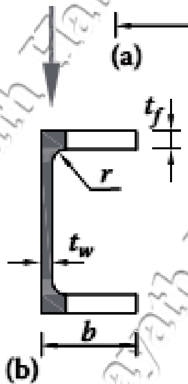
The design shear resistance of a cross-section is denoted by $V_{c,Rd}$ and may be calculated based on a plastic ($V_{pl,Rd}$) or an elastic distribution of shear stress.

A_v is the shear area.

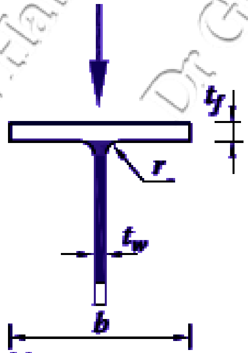
$$V_{pl,Rd} = \frac{A_v f_y}{\gamma_{M0}\sqrt{3}}$$



a) rolled I and H sections, load parallel to web $=A-2t_f b+(t_w+2r)t_f \geq \eta h_w t_w$



b) rolled channel sections, load parallel to web $=A-2t_f b+(t_w+r)t_f$

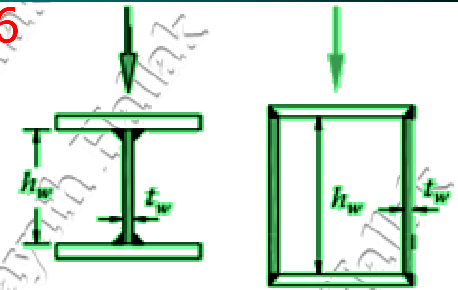


c) rolled T-section, load parallel to web $=0.9\,(A-b\,t_f)$

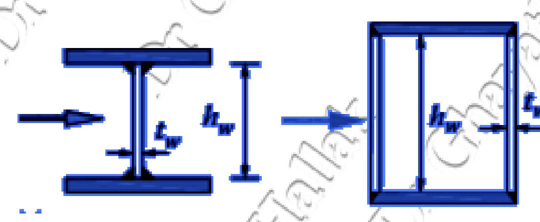
Shear resistance $V_{c,Rd}$

EN 1993-1-1: 2005- 6.2.6

d) welded I, H and box sections, load parallel to web $=\eta\sum(h_w t_w)$



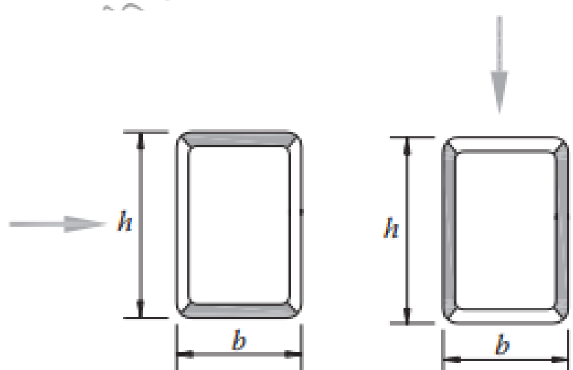
e) welded I, H, channel and box sections, load parallel to flanges $=A-\sum(h_w t_w)$



f) rolled rectangular hollow sections RHS of uniform thickness:

load parallel to depth $=Ah/(b+h)$

load parallel to width $=Ab/(b+h)$



g) Circular hollow sections CHS and tubes of uniform thickness $=2A/\pi$

$\eta = 1,0$.

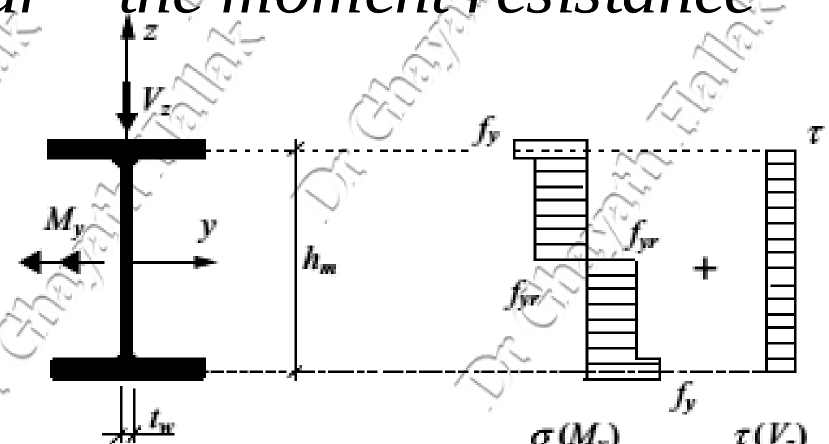
Moment resistance $M_{c,Rd}$ EN 1993-1-1: 2005- 6.2.5

$$1 - V_{Ed} \leq 0.5 V_{pl,Rd} \Rightarrow Low\ Shear\ (No\ effects\ on\ Moment\ Resistance)$$

Section classification	Bending
Class 1, 2	$M_{c,Rd}=M_{pl,Rd}=W_{pl} f_y/\gamma_{M0}$
Class 3	$M_{c,Rd}=M_{el,Rd}=W_{el} f_y/\gamma_{M0}$
Class 4	$M_{c,Rd}=W_{eff} f_y/\gamma_{M0}$

$$2 - V_{Ed} > 0.5 V_{pl,Rd} \Rightarrow High\ Shear - the\ moment\ resistance$$

should be calculated using reduced yield strength for the ***shear area***, *given by :*

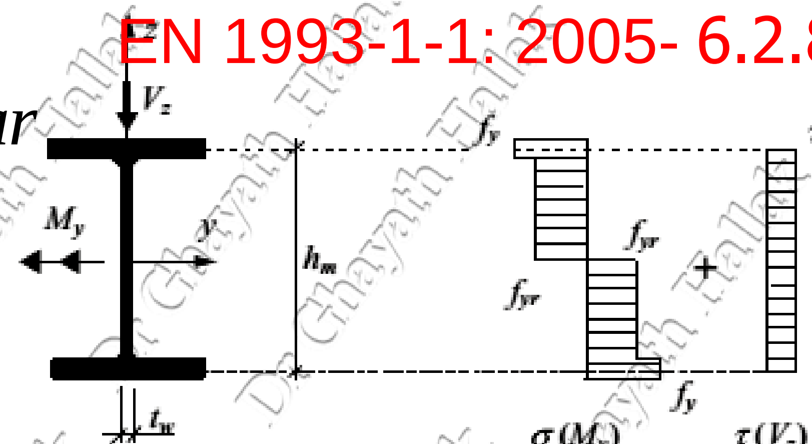


Moment resistance $M_{c,Rd}$

EN 1993-1-1: 2005- 6.2.8

$$2 - V_{Ed} > 0.5\, V_{pl,Rd} \Rightarrow High\ Shear$$

$$f_{yr} = (1-\rho) f_y \ , \rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1 \right)^2$$



For I sections with equal flanges and bending about the major axis, the reduced design plastic resistance moment allowing for the shear force effect is given by

$$M_{y,v,Rd} = \frac{\left(W_{pl,y} - \rho \dfrac{A_w^2}{4t_w} \right) f_y}{\gamma_{M0}} \leq M_{y,C,Rd} \longleftrightarrow \left(\begin{array}{l} according\ to\ the\ class \\ of\ the\ cross\ section \end{array} \right)$$

$$M_{y,v,Rd} = \frac{\left(W_{pl,y} \right) f_{yr}}{\gamma_{M0}} (more\ conservative) \leq M_{y,C,Rd}$$

Moment resistance $M_{c,Rd}$ EN 1993-1-1: 2005- 6.2.5

$$Where: W_{pl,y} = bt_f h_w + \frac{A_w^2}{4t_w},\ f_{yr} = (1-\rho) f_y\ ,\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2$$

Shear Buckling $V_{b,Rd}$

Shear Buckling $V_{b,Rd}$ can be ignored if:

$$\frac{h_w}{t_w} \le 72\frac{\varepsilon}{\eta}\left(for\ unstiffened\ webs\right), where \quad \eta = 1, \varepsilon = \sqrt{\frac{235}{f_y}}$$

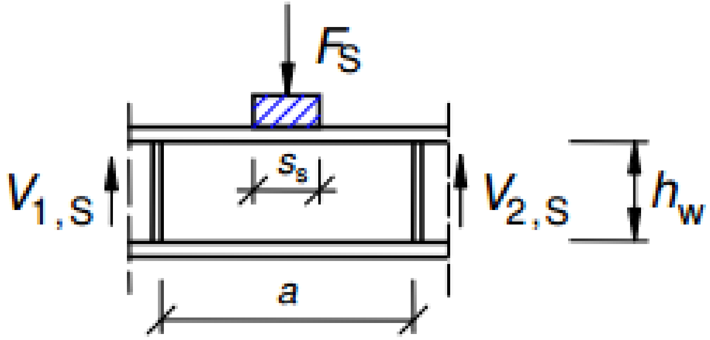
$$\frac{h_w}{t_w} \le \frac{31}{\eta}\varepsilon\sqrt{k_\tau}\left(for\ stiffened\ webs\right)$$

RESISTANCE OF THE BEAM TO TRANSVERSE LOADING

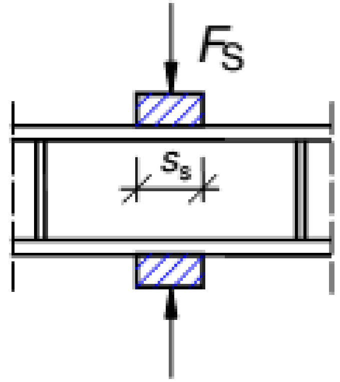
Transverse loading denotes a load that is applied perpendicular to the flange in the plane of the web.

EN 1993-1-5, (assume that the compression flange has an adequate lateral and torsional restraint) covers three types of transverse load applied through a flange to the web:

1. Load application through one flange and resisted by shear forces in the web

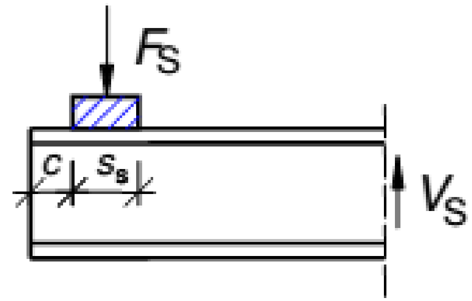


2. Load application through one flange and transferred through the web directly to the other flange



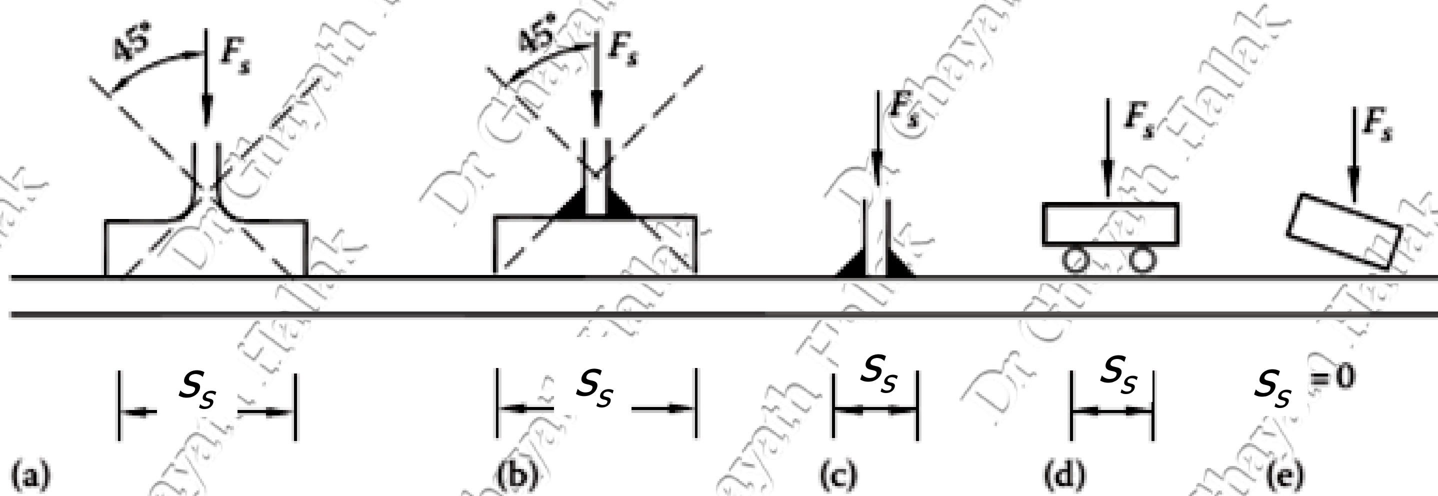
RESISTANCE OF THE BEAM TO TRANSVERSE LOADING

3. Load application through one flange adjacent to an unstiffened end.



1- Length of stiff bearing

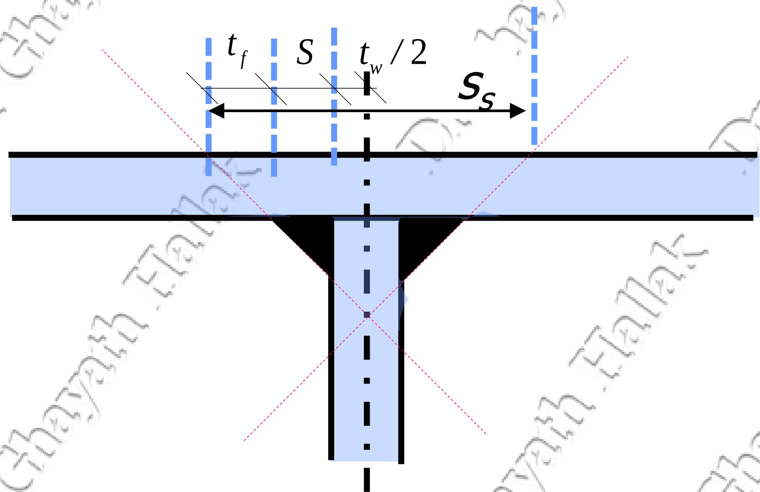
The length of stiff bearing s_s corresponds to the loaded length on top of the flange. s_s should not be taken as larger than h_w.



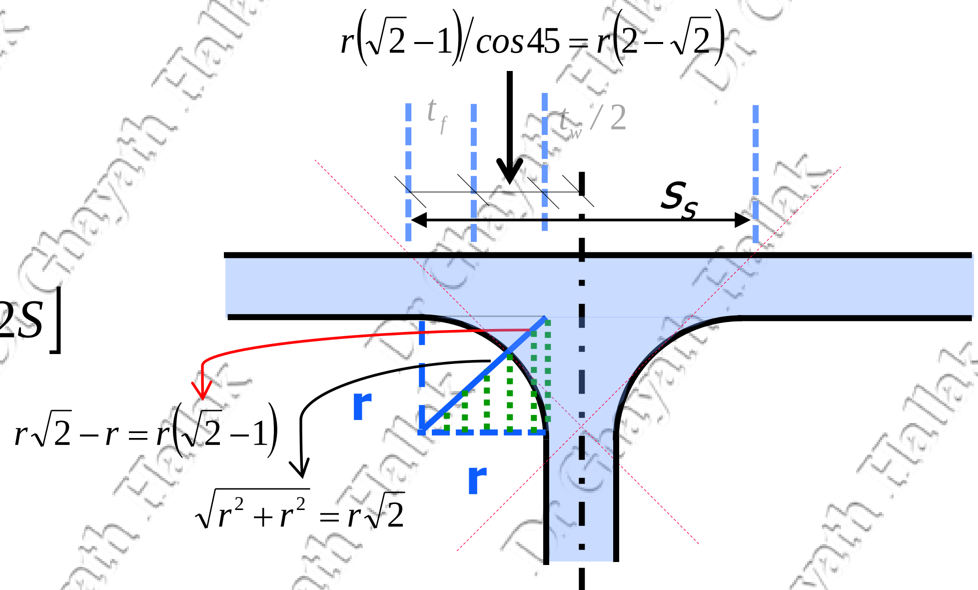
Length of stiff bearing: (a) rolled section; (b) welded section; (c) plate directly; (d) crane wheel; (e) no deformation of girder.

RESISTANCE OF THE BEAM TO TRANSVERSE LOADING

1- Length of stiff bearing



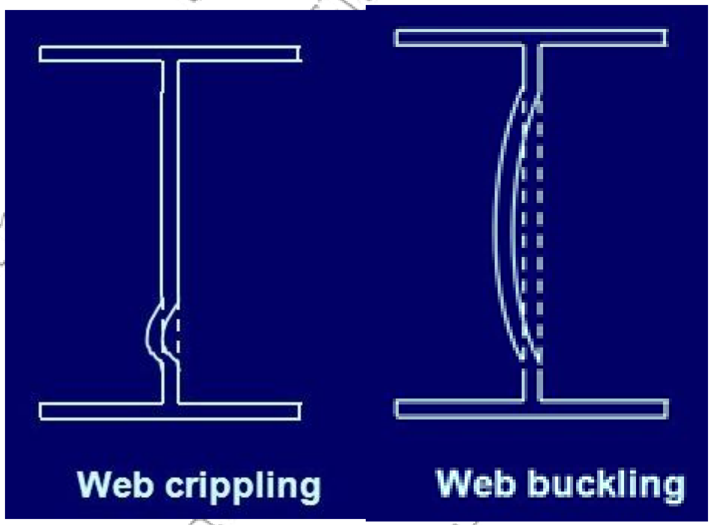
$$S_s = 2\left[t_f + t_w/2 + S\right] = \left[2t_f + t_w + 2S\right]$$



$$S_s = 2\left[t_f + t_w/2 + r\left(2-\sqrt{2}\right)\right] = \left[2t_f + t_w + 2r\left(2-\sqrt{2}\right)\right]$$

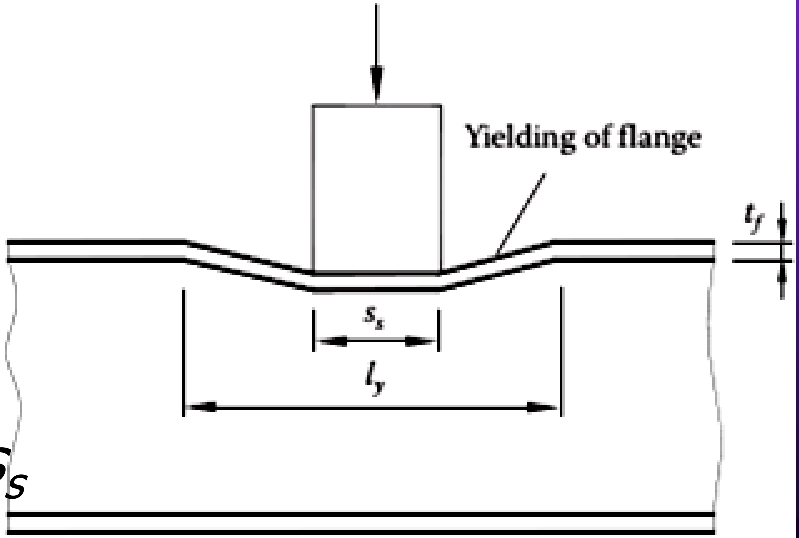
RESISTANCE OF THE BEAM TO TRANSVERSE LOADING

2 Design resistance



For unstiffened or stiffened webs, the design resistance to local buckling under transverse loading should be taken as

$$F_{Rd} = \frac{f_{yw} L_{eff} t_w}{\gamma_{M1}} = \chi_F \frac{f_{yw} l_y t_w}{\gamma_{M1}}$$



χ_F is the reduction factor due to local buckling

l_y is the effective loaded length appropriate to the length of the stiff bearing s_s

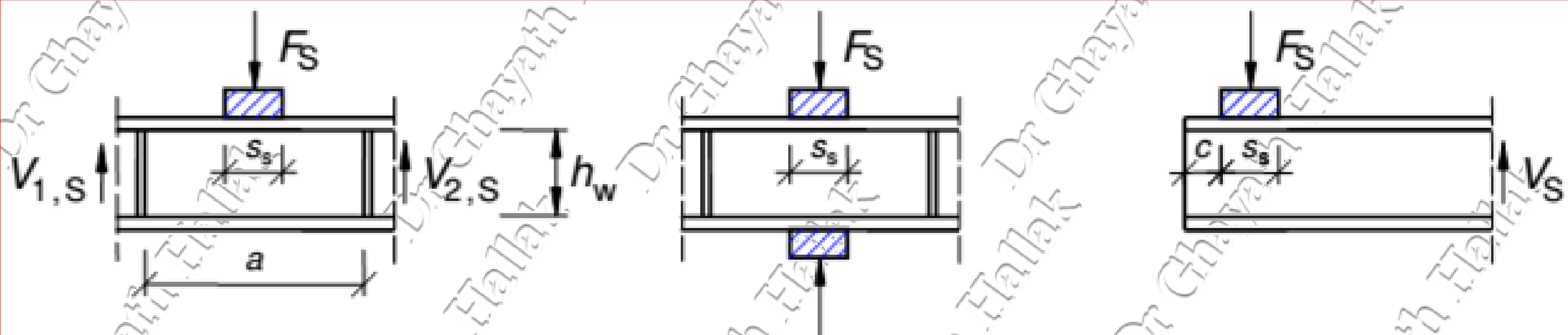
RESISTANCE OF THE BEAM TO TRANSVERSE LOADING

3 Reduction factor χ_F for effective length for resistance

$$\chi_F = \frac{0.5}{\bar{\lambda}_F} \leq 1.0, \quad \bar{\lambda}_F = \sqrt{\frac{l_y\, t_w\, f_{yw}}{F_{cr}}}, \quad F_{cr} = 0.9\, K_F\, E\, \frac{t_w^3}{h_w}$$

F_{cr} is the elastic critical load of the web

k_F is the buckling value for transverse loading



$$k_F = 6 + 2\left(\frac{h_w}{a}\right)^2 \qquad k_F = 3{,}5 + 2\left(\frac{h_w}{a}\right)^2 \qquad k_F = 2 + 6\left(\frac{s_s + c}{h_w}\right) \leq 6$$

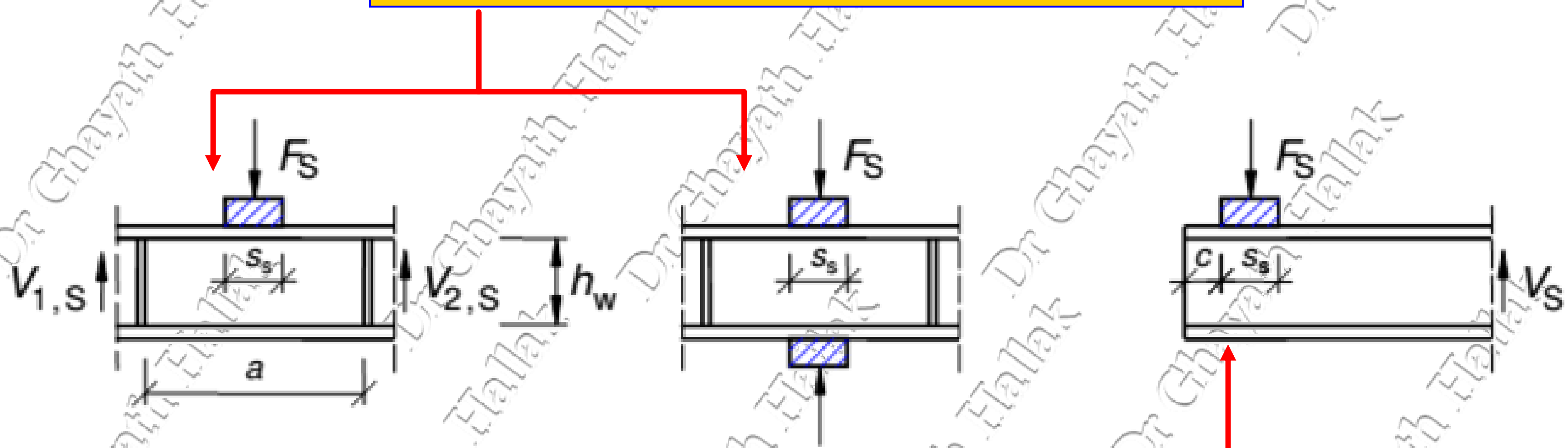
Buckling coefficients for webs without longitudinal stiffeners

RESISTANCE OF THE BEAM TO TRANSVERSE LOADING

4 Effective loaded length-EN1993-1-5 §6.5

The effective loaded length l_y corresponds to the effective loaded length of the web taking into account the influence of the flange.

$$l_y = S_s + 2t_f\left(1 + \sqrt{m_1 + m_2}\right) \le a$$



$$l_y = min\left(l_y, l_{y1}, l_{y2}\right)$$

RESISTANCE OF THE BEAM TO TRANSVERSE LOADING

4 Effective loaded length-EN1993-1-5 §6.5

$$l_{y1} = l_e + t_f \sqrt{\frac{m_1}{2} + \left(\frac{l_e}{t_f}\right)^2 + m_2} \;, \qquad l_{y2} = l_e + t_f \sqrt{m_1 + m_2}$$

$$l_e = \frac{k_F E t_w^2}{2 f_{yw} h_w} \le S_s + C, \qquad m_2 = 0 \quad for\ \overline{\lambda_F} \le 0.5$$

$$m_1 = \frac{f_{yf} b_f}{f_{yw} t_w}, \qquad m_2 = 0.02 \left(\frac{h_w}{t_f}\right)^2 \quad for\ \overline{\lambda_F} > 0.5$$

The **verification** of a beam subject to transverse loading is

$$\frac{F_{Ed}}{F_{Rd}} \le 1.0, \; F_{Ed} \text{ is the design transverse loading.}$$

Deflections

When designing according to limit state principles it is customary to check that deflections at **working load levels** will not impair the proper function of the structure.

Excessive serviceability deflections include:

- ❑- cracking of plaster ceilings
- ❑- allowing crane rails to become misaligned
- ❑- causing difficulty in opening large doors.

In general, beam deflection is a function of:

- ➢- the span length
- ➢- end restraints
- ➢- modulus of elasticity of the material
- ➢- moment of inertia of the cross section
- ➢- loading

Deflections

Deflection checks should be made against unfactored permanent actions and unfactored variable actions. However, Clause NA.2.23 of NA to BS EN 1993-1-1 proposes that **permanent actions** should be taken as **zero** in Deflection checks. The Deflection loads are just the **unfactored variable actions (combination).**

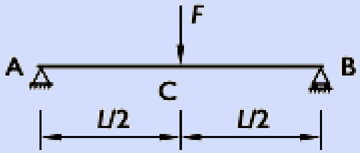
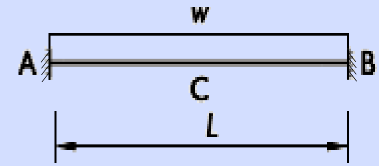
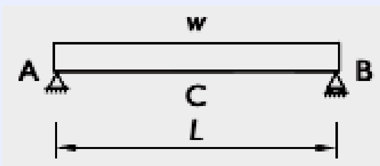
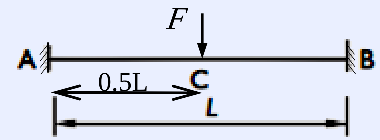
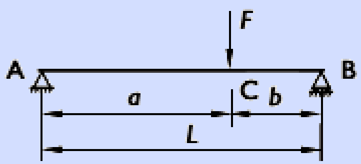
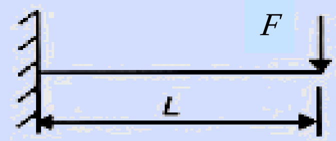
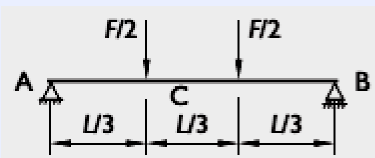
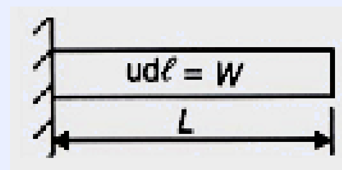
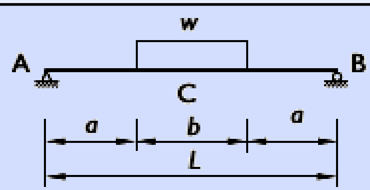
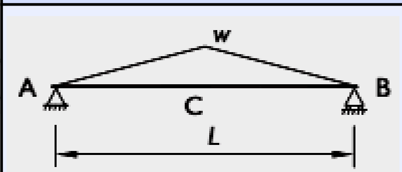
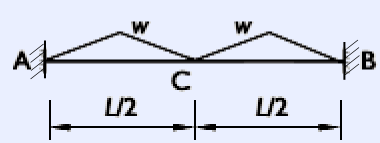
impose Q_I, Wind Q_W

comb1=Q_I+0.5 Q_W , comb2=0.7Q_I+ Q_W

Deflections for some common load cases for simply supported beams, both end fixed beam and cantilever beam, are given in the following Table:

The maximum deflection calculated must not exceed the deflection limit. The deflection limits are not given directly in Eurocode 1993, instead, reference must be made to the National Annex.

Deflection of beams under various loads

Loading	Deflection	Loading	Deflection
	$\frac{FL^3}{48EI}$		$\frac{WL^4}{384EI}$
	$\frac{5WL^4}{384EI}$		$\frac{FL^3}{192EI}$
	$\frac{F\,b\,(3L^3-4b^4)}{48EI}$		$\frac{FL^3}{3EI}$
	$\frac{6.81FL^3}{384EI}$		$\frac{WL^3}{8EI}$
	$\frac{W\,b\,(8L^3-4\,L\,b^2+b^3)}{384EI}$		
	$\frac{WL^4}{120EI}$		$\frac{WL^4}{146.3EI}$

Vertical Deflection Limits from NA 2.23 Clause 7.2.1(1) B

Design Situation	Deflection limit
Cantilever	Length/180
Beams carrying plaster of other brittle finish	Span/360
Other beams (except purlins and sheeting rails)	Span/200
Purlins and sheeting rails	To suit the characteristics of particular cladding

Horizontal Deflection Limits from NA 2.24

Design Situation	Deflection limit
Tops of columns in single storey buildings, except portal frames	Height/300
Columns in portal frame buildings, not supporting crane runways	To suit cladding
In each storey of a building with more than one storey	Height of storey/300